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## Basic review

Proportions use ratios. A proportion is a statement of equality between two ratios. A proportion can be written as:

$$\frac{a}{b} = \frac{c}{d}$$

Proportions are typically used when you want to solve for an unknown. Let's look back to our car example. In the last section we found we could drive 120 miles on 4 gallons of gas. We want to find out how many miles we could drive on 10 gallons of gas. This information is displayed in the table below.

| | Miles Traveled | Gallons of Gas |
|---|---|---|
| Last Week's Trip | 120 | 4 |
| Next Trip | $x$ | 10 |

The value we want to determine is represented by an $x$ in the table above. We can find this value by setting up a proportion. This is shown below.

$$\frac{120 \text{ miles}}{4 \text{ gallons}} = \frac{x \text{ miles}}{10 \text{ gallons}}$$

Once we set up the proportion, we want to determine the value of $x$.

To solve this proportion for one unknown, multiply both sides of the proportion by the denominator of the fraction that contains the $x$. This can be done because if we multiply both sides of an equation (in this case, written as a proportion) by the same number, they are still equal.

If we use the proportion above, our unknown is $x$. To solve for this unknown:

Multiply both sides by the denominator of the fraction containing the unknown.

$$10\times\frac{120}{4}=\frac{x}{10}\times 10 \quad \text{or} \quad \frac{1200}{4}=x$$

Simplify this fraction.

$$x=\frac{1200}{4}=\frac{300\times 4}{4}=300$$

$$x=300$$

Here we can see if we have 10 gallons of gas, we can drive 300 miles.

| | Miles Traveled | Gallons of Gas |
|---|---|---|
| Last Week's Trip | 120 | 4 |
| Next Trip | 300 | 10 |

From the discussion above, we can see that proportions use ratios as a way to solve for an unknown. When you want to solve for an unknown using proportions, you should follow these steps:

1. Set up a table of information to determine what we know and what we want to find.
2. Use the information in the table to set up a proportion.
3. Multiply both sides of the proportion by the denominator of the fraction containing the unknown.
4. Simplify the result.

**Example**
We go to the store and purchase a 5-pound bag of peanuts for $2.10. Assuming that the price per pound doesn't change, how much will a 7-pound bag of peanuts cost?

1. Set up a table of information to determine what we know and what we want to find.

| | Cost | Pounds |
|---|---|---|
| First Bag | $2.10 | 5 |
| Second Bag | $x$ | 7 |

2. Use the information in the table to set up a proportion.

$$\frac{2.10}{5} = \frac{x}{7}$$

3. Multiply both sides of the proportion by the denominator of the fraction containing the unknown.

$$7 \times \frac{2.10}{5} = \frac{x}{7} \times 7 \quad \text{or} \quad \frac{14.7}{5} = x$$

4. Simplify the result. To simplify this equation, we need to divide 14.7 by 5.

$$\begin{array}{r} 2.94 \\ 5\overline{)\,14.70} \\ -10 \\ \hline 47 \\ -45 \\ \hline 20 \\ -20 \\ \hline 0 \end{array} \quad \text{so, } x = 2.94$$

Here we can see the 7-pound bag would cost $2.94.

| | Cost | Pounds |
|---|---|---|
| First Bag | $2.10 | 5 |
| Second Bag | $2.94 | 7 |

## Proportions and percents

One type of problem where proportions can be particularly useful is in problems involving percents. When solving percent problems, it helps to set up proportions dealing with percents as two ratios that compare part to whole. If you use our table format above, our table would look like:

| | Percent | Number of Cases |
|---|---|---|
| Part of Group | percent | part of case |
| Whole Group | 100 | whole case |

From this table, we can see that the proportion we could set up is:

$$\frac{\text{percent}}{100} = \frac{\text{part of case}}{\text{whole of case}}$$

If we are given that 25% of the 16 real estate companies around central New York have closed their businesses, we can use proportions to determine the number that closed. For this example, we are given the percent as 25%, and the whole in this case is 16.

(**NOTE:** the wording 25% of 16 tells you that the whole is 16.) If you use our table format above, our table would look like:

| | Percent | Number of Cases |
|---|---|---|
| Part of Group | 25 | $x$ |
| Whole Group | 100 | 16 |

If we use these numbers in the equation above, we get:

$$\frac{25}{100} = \frac{x}{16}$$
$$16 \times \frac{25}{100} = x$$
$$\frac{400}{100} = x$$
$$4 = x$$

We see that 4 of the real estate companies around central New York have closed their businesses. This can also be presented as a fraction. When you come across problems such as these, the important thing to keep in mind is that both proportions are part/whole.

**Example**

In 1996, 30% of the 2100 car buyers at Faketown Auto Dealers financed through the dealership. How many car buyers financed through the dealership?

**NOTE**: The question is really asking: what is 30% of 2100?

Set up a table of information to determine what we know and what we want to find.

| | Percent | Number of Cases |
|---|---|---|
| Part of Group | 30 | $x$ |
| Whole Group | 100 | 2,100 |

Use the information in the table to set up a proportion.

$$\frac{30}{100} = \frac{x}{2100}$$

Multiply both sides of the proportion by the denominator of the fraction containing the unknown.

$$2100 \times \frac{30}{100} = x$$
$$\frac{63000}{100} = x$$

Simplify the result.

630 = *x*

We find that of the 2100 car buyers at Faketown Auto Dealers, 630 of them financed through the dealership.

**Converting Fractions to Percents Using Proportions**

In the section on percents, we discussed one method of converting fractions to percents. We showed that one way of converting . to a percent was to convert the fraction to a decimal (divide . to get .25) and then convert the decimal to a percent (multiply .25 by 100 to get 25%).

Converting fractions to percents can also be done using proportions.
To do this, set the fraction = to x/100 and solve for x. For example,

$$\frac{1}{4} = \frac{x}{100}$$
$$100 = 4x \text{ or } \frac{100}{4} = x$$
$$x = 25$$

## Proportions and basic rates

Another type of problem that uses proportions is the basic rate problem. An example of this kind of problem is:

The temperature dropped 15 degrees in the last 30 days. If the rate of temperature drop remains the same, how many degrees will the temperature drop in the next ten days?

| | Temperature Drop | Number of Days |
|---|---|---|
| Initial Drop | 15 | 30 |
| New Drop | x | 10 |

By looking at the table above, we should be able to see a proportion we can set up. One proportion is shown below.

$$\frac{15 \text{ degrees}}{30 \text{ days}} = \frac{x \text{ degrees}}{10 \text{ days}}$$

Once this is done, all we need to do is solve for *x*.

$$10 \times \frac{15}{30} = x$$

$$5 \text{ degrees} = x$$

**NOTE:** You could set up another proportion using the same information and yielding the same result. That proportion is shown below.

$$\frac{10 \text{ days}}{30 \text{ days}} = \frac{x \text{ degrees}}{15 \text{ degrees}}$$

Both proportions shown above are correct. In the first proportion we set up, we used Degrees/Days ratio. In the second we used a part/whole ratio. The important thing to remember is to be consistent with both ratios you set up in a proportion.

**Example**

The stock market rose 80.0 points in the last 3 days. If this rate of increase continues, how much will the stock market rise over the next 5 days? (Report your answer to the nearest tenth.)

1. Set up a table of information to determine what we know and what we want to find.

| | Rise in Points | Number of Days |
|---|---|---|
| First Rise | 80 | 3 |
| New Rise | $x$ | 5 |

2. Use the information in the table to set up a proportion.

$$\frac{80 \text{ points}}{3 \text{ days}} = \frac{x \text{ points}}{5 \text{ days}}$$

3. Multiply both sides of the proportion by the denominator of the fraction containing the unknown.

$$5 \times \frac{80}{3} = x$$

$$\frac{400}{3} = x$$

4. Simplify the result.

$133.3 = x$

At the same rate of increase, the market will rise 133.3 points over the next 5 days.